1. Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 inches2 in t minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3 second time interval. A reasonable choice of distribution is P
2. Let X be a random day of the week, coded so that Monday is 1, Tuesday is 2, etc. (so X takes values 1, 2,..., 7, with equal probabilities). Let Y be the next day after X (again represented as an integer between 1 and 7). Do X and Y have the same distribution? What is P(X)

Answer:

1. The number of raindrops hitting a particular region measuring 5 inches^2 in t minutes can be modeled using a Poisson distribution. The Poisson distribution is commonly used to model events that occur randomly over time or space, given a known rate of occurrence. In this case, raindrops are falling randomly over a certain area at an average rate of 20 drops per square inch per minute.

Let λ be the expected number of raindrops that hit the 5-inch^2 region in t minutes. Then λ = (20 drops/inch^2/minute) × (5 inch^2) × (t minutes) = 100t. Thus, the Poisson parameter is λ = 100t.

The probability of having no raindrops in a given 3-second time interval can be calculated as follows:

Let X be the number of raindrops in a 3-second time interval. Then X has a Poisson distribution with parameter λ' = λ/20, since there are 20 × 3 = 60 3-second intervals in a minute.

The probability of having no raindrops in a 3-second time interval is given by P(X=0) = e^(-λ') \* (λ')^0 / 0! = e^(-λ') = e^(-(λ/20)) = e^(-(100t/20)) = e^(-5t).

1. X and Y do not have the same distribution. While X is uniformly distributed over {1, 2, ..., 7}, Y is not uniformly distributed. If X = 7, then Y = 1 with probability 1, while if X is any other day of the week, Y is uniformly distributed over the remaining days.

The probability distribution of X is given by:

P(X = i) = 1/7, for i = 1, 2, ..., 7.

To see this, note that there are 7 equally likely outcomes for the day of the week.

The probability distribution of Y given X = i is given by:

P(Y = j | X = i) = 1/6, for j ≠ i, j = 1, 2, ..., 7.

To see this, note that there are 6 possible outcomes for Y given X = i, and they are all equally likely.